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THE MATHEMATICAL MODEL OF THE OPERATION PROBLEM PRESENTED ON A PARTICULAR MODEL

Abstract: The aim of the thesis is to design a model of closed system M/G/1 for knitting production in the Svitex company. I strive to determine specific values of the machine idle time. The Poisson process for the occurrence of disturbances was used. This is consistent with the proposed pair of compatible hypotheses. The procedure described has proven to be correct. After postulation of the hypotheses, a specific closed system was proposed. When applying to a particular situation, a result using the equations and normative condition was elaborated.

Key words: mathematical model, Poisson process, mathematics operations.

JEL Classification: C20, C21

Introduction

As Galileo Galilei already said, the book of Nature is written in the language of mathematics. Martin Heidegger, in turn, "characterizes the mathematical nature as being exact, which means the link to the subject circuit in that it is a time-domain definition of velocities in motion, measurable through numbers and counting" (Ambrozy 2012, p. 264). Mathematics is also a great tool for analyzing production processes, which is reflected in the impact on the economy. At this point, I strive to create a closed M / G / 1 system for the Svitex company involved in knitting production and to define a non-productive period of idle time.

To fulfil the aim, the following steps should be done:

1 to develop the theoretical background to the issue,

2 to get data from the enterprise,

3 to analyze the current state according to the obtained data,

4 to address the current situation using statistical methods,

5 to evaluate the situation.

To meet the objectives of the problem, there were used empirical, statistical analytical methods of work, modelling and comparison. Empirical methods as observation and measurement were applied to obtain business data. When analyzing the current state, I used the χ^2 statistical method of good match to determine if the input flow of requirements is the Poisson process. Subsequently, a closed M / G / 1 model is created as the system has met the necessary conditions. In the "Proposals and Measures" chapter there was used a comparison of the current state with the models proposed.

The χ^2 test of good match is more detailed. By testing statistical hypotheses, verification of the accuracy of the claim is done. The hypothesis is a certain claim that is not underpinned yet. In this case, it relates to the probability distribution of random variables by testing the consistency of empirical and theoretical distribution. The hypothesis that was validated is a zero hypothesis. It is marked with H_0. Its negation is the hypothesis H_1. The conclusion of the test is the decision which hypothesis is accepted and which is refused.

χ^2 good match test is also called the Pearson test. It tests for a certain probability distribution the following pair of hypotheses:

H_0: empirical frequencies in each category are equal to theoretical frequencies; vice versa

H_1: empirical frequencies in each category are not equal to theoretical frequencies.

Empirical frequencies in this case represent the number of inputs of system requirements per day, i.e. how many times the machine needs a certain repair done by worker per day. Theoretical frequencies on the other hand are derived from Poisson division (Kingman 1992). If the difference is not statistically significant, that is, it is small enough, I will not refuse the zero hypothesis, I will accept it, i.e. inputs to the system are Poisson-like.

The test criterion (statistics) is $$\chi^2 = \sum_{i=1}^{k} \frac{(n_i - n.p_i)^2}{n.p_i} \qquad (8)$$

where

n_i - are empirical frequencies,

p_i - are theoretical frequencies of probabilities based on the theoretical probability distribution - Poisson distribution,

n - is the size of the file whose elements are being examined,

k - is the number of the categories into which the elements are divided - the number of machine inputs into the system per day.

If the zero hypothesis is valid then $\chi^2 \approx \chi^2$ (r-1-p), where r is the number of intervals and p is the number of estimated parameters of a given division (for the Poisson distribution, the number of estimated parameters is p = 1). This means that the zero hypothesis - the choice stems from the Poisson division - applies if this variable has an asymptotically chi-squared distribution of r - 2 degrees of freedom. If $\chi^2 \geq \chi^2_{r-2}(\alpha)$, I reject a zero hypothesis at a level that is asymptotically equal to α.

"In the Pearson test, the so-called Cohran rule (Plackett 1983) should be applied consequently. The condition for using the test is $n.p_i \geq 5$ for all i = 1, 2,. . . , k. This condition is sometimes difficult to follow in practice. However, strict adherence to it is necessary only in the few degrees of freedom of test statistic χ2. It has been verified that for the number of inputs r-1-p ≥ 3, it is sufficient for $n.p_i \geq 4$ and for r-1-p ≥ 6 to suffice $n.p_i \geq 1$. If these conditions are not met, it is recommended to merge the adjacent intervals with small number. (Ostertagová, 2011)"

The Svitex company is an enterprise active in the textile industry. The main business is textiles, clothing and leather production. Production focuses mainly on the production of knitted ladies' stockings, socks and hosiery.

The BUZI knitting machine will be focused on in the thesis. There are more than 200 of these machines in the workshop. One worker - operator serves 35 machines. Their role is to rework machines for new designs (new products), quality control ("moulding"), continuous maintenance (e.g. "lubrication" of machines) and repair of faults. The disorders that may occur are e.g. electric problems, tearing in the heel, tear-off core cane, skidding, broken needle, erased programme, broken tightening up, need to replace the seals, run eyes, jerks, and so on.

Modelling the occurrence of failures on BUZI machines as a Poisson process.

First, it was needed to determine how often failures occur on individual machines. By observing, measuring, and following calculations, it was found that the average number of failures per day per machine was 0.60. This figure expresses the number of inputs to the system, i.e. λ = 0.60. The number of requests that run in the system is the number of machines that is handled by an operator, i.e. m = 35. Then it was needed to find out whether the inputs of the requests represent the Poisson process. The χ^2good match test has been used. The hypotheses have been formulated:

H_0: empirical frequencies in each category are equal to theoretical frequencies, i.e. they are the Poisson division;

vice versa

H_1: empirical frequencies in each category are not equal to theoretical frequencies.As I have already mentioned, I did not recognize Poisson distribution, so I approximated it with a point estimate. Due to this, the sample mean is $\lambda \approx \bar{k}$. The measured data are shown in Table 1, where k is the number of failures that occurred on one machine in one day, and n_i is how many times the situation occurred on thirty-five machines during eighty-nine days. Using the table, this parameter was calculated from the relationship

$$\bar{k} = \frac{1}{n}\sum_{i=0}^{6} k.n_i$$ k=0,60, n= 1,038

Table 1. **Empirical number of failures**

k	n_i	$k.n_i$
0	589	0
1	320	318
2	97	194
3	25	75
4	5	28
5	0	0
6	2	12
∑	1.038	627

Source: Own processing

For the calculation of the test statistics, the auxiliary Table 2 was prepared. The first column contains the k values that express the number of times the machine enters the system a day. For these needs, I have merged intervals for k = 4, 5, 6, as they exhibited low abundance. It was also necessary to meet Cochran's rule. There has been reduced the number of degrees of freedom from 7 to 5 (k - 1 - p = 7 - 1 - 1 = 5) and then n.p(k) ≥ 3. In the second column, according to relationship there are calculated theoretical probabilities p(k) = $\frac{\lambda^k}{k!}.e^{-\lambda}$. In the third column, the theoretical number of Poisson disturbances is calculated as n.p(k). In the fourth column there are inserted empirical abnormalities of malfunctions n_i. In the fifth column, the difference in empirical and theoretical probabilities is increased to the second, which was used in relation for the calculation $\chi^2 = \sum_{i=1}^{k}\frac{(n_i - n.p(k))^2}{n.p(k)}$. The partial calculations are in the last column, where the last row shows the test characteristic χ^2 = 6.883.

Table 2. **Calculation of test characteristic χ^2**

k	p(k)	n. p(k)	n_i	$(n_i - n.p(k))^2$	χ^2
0	0.5488116	569.666	589	373.785063	0.656
1	0.329287	341.8	320	475.235071	1.39
2	0.0987861	102.54	97	30.6912243	0.299
3	0.0197572	20.508	25	20.1781249	0.984
4 až 6	0.0033548	3.48226	7	12.3745141	3.554
$\sum$	0.9999967				**6.883**

Source: Own processing

From the statistical tables, it was found that the critical value $\chi^2_{(0,05,\ 3)} = 7.81$ (Markechová, Tirpáková and Stehlíková, 2011, p. 361). The condition $\chi^2 \leq \chi^2_{(0,05,\ 3)}$ is fulfilled, so the hypothesis H_0 was accepted. Thus, the distribution of malfunctions is Poisson division. Comparison of empirical and theoretical abnormalities on BUZI machines is also illustrated in Figure 1.

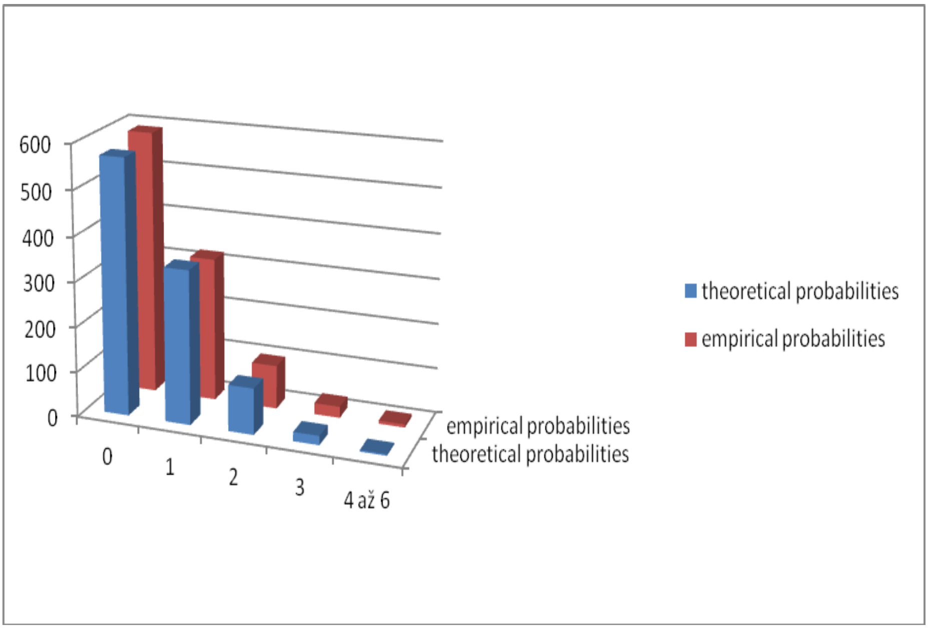

Figure 1. **Empirical and theoretical abnormalities on BUZI machines**

Source: Own processing

Modelling of closed system M / G / 1

With regard to the fact that in this situation, one worker is assigned to operate 35 same BUZI machines, supervising and performing certain operations on them, and also requirements occurring at random times, i.e. this corresponds to the Poisson division, so this situation corresponds to the M / G / 1 model - compare (Yang et al., 1994).

The next task is to determine the mean stoppage of the machine in the queue, i.e. unproductive machine downtime and use of employee's working time.

To solve the example, it is still needed to know the average value of the service duration τ. It was calculated as the average time of the operator's service, which came out 22 minutes. Since the machines work continuously, i.e. during three shifts and I count with one worker per one shift, after the separation, the duration of service is 7.3 minutes, which is 0.12 h = τ.

Now I need to calculate P_0 using the systems of equations and normative condition. For this assignment, it is a set of thirty-five equations (due to their extensiveness, I do not present them all) :

$$P_o = (P_o + P_1)\, \pi_{1,0}$$
$$P_1 = (P_o + P_1)\, \pi_{1,1} + P_2\, \pi_{2,0}$$
$$P_2 = (P_o + P_1)\, \pi_{1,2} + P_2\, \pi_{2,1} + P_3\, \pi_{3,1}$$
$$P_3 = (P_o + P_1)\, \pi_{1,3} + P_2\, \pi_{2,2} + P_3\, \pi_{3,1} + P_4\, \pi_{4,0}$$
$$P_4 = (P_o + P_1)\, \pi_{1,4} + P_2\, \pi_{2,3} + P_3\, \pi_{3,2} + P_4\, \pi_{4,1} + P_5\, \pi_{5,0}$$
$$P_5 = (P_o + P_1)\, \pi_{1,5} + P_2\, \pi_{2,4} + P_3\, \pi_{3,3} + P_4\, \pi_{4,2} + P_5\, \pi_{5,1} + P_6\, \pi_{6,0}$$

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$$P_{30} = (P_o + P_1)\, \pi_{1,30} + P_2\, \pi_{2,29} + P_3\, \pi_{3,28} + P_4\, \pi_{4,27} + P_5\, \pi_{5,26} + P_6\, \pi_{6,25} + P_7\, \pi_{7,24} + P_8\, \pi_{8,23} + P_9\, \pi_{9,22} + P_{10}\, \pi_{10,21} + P_{11}\, \pi_{11,20} + P_{12}\, \pi_{12,19} + P_{13}\, \pi_{13,18} + P_{14}\, \pi_{14,17} + P_{15}\, \pi_{15,16} + P_{16}\, \pi_{16,15} + P_{17}\, \pi_{17,14} + P_{18}\, \pi_{18,13} + P_{19}\, \pi_{19,12} + P_{20}\, \pi_{20,11} + P_{21}\, \pi_{21,10} + P_{22}\, \pi_{22,9} + P_{23}\, \pi_{23,8} + P_{24}\, \pi_{24,7} + P_{25}\, \pi_{25,6} + P_{26}\, \pi_{26,5} + P_{27}\, \pi_{27,4} + P_{28}\, \pi_{28,3} + P_{29}\, \pi_{29,2} + P_{30}\, \pi_{30,1} + P_{31}\, \pi_{31,0}$$

$$P_{31} = (P_o + P_1)\, \pi_{1,31} + P_2\, \pi_{2,30} + P_3\, \pi_{3,29} + P_4\, \pi_{4,28} + P_5\, \pi_{5,27} + P_6\, \pi_{6,26} + P_7\, \pi_{7,25} + P_8\, \pi_{8,24} + P_9\, \pi_{9,23} + P_{10}\, \pi_{10,22} + P_{11}\, \pi_{11,21} + P_{12}\, \pi_{12,20} + P_{13}\, \pi_{13,19} + P_{14}\, \pi_{14,18} + P_{15}\, \pi_{15,17} + P_{16}\, \pi_{16,16} + P_{17}\, \pi_{17,15} + P_{18}\, \pi_{18,14} + P_{19}\, \pi_{19,13} + P_{20}\, \pi_{20,12} + P_{21}\, \pi_{21,11} + P_{22}\, \pi_{22,10} + P_{23}\, \pi_{23,9} + P_{24}\, \pi_{24,8} + P_{25}\, \pi_{25,7} + P_{26}\, \pi_{26,6} + P_{27}\, \pi_{27,5} + P_{28}\, \pi_{28,4} + P_{29}\, \pi_{29,3} + P_{30}\, \pi_{30,2} + P_{31}\, \pi_{31,1} + P_{32}\pi_{32,0}$$

$$P_{32} = (P_o + P_1)\, \pi_{1,32} + P_2\, \pi_{2,31} + P_3\, \pi_{3,30} + P_4\, \pi_{4,29} + P_5\, \pi_{5,28} + P_6\, \pi_{6,27} + P_7\, \pi_{7,26} + P_8\, \pi_{8,25} + P_9\, \pi_{9,24} + P_{10}\, \pi_{10,23} + P_{11}\, \pi_{11,22} + P_{12}\, \pi_{12,21} + P_{13}\, \pi_{13,20} + P_{14}\, \pi_{14,19} + P_{15}\, \pi_{15,18} + P_{16}\, \pi_{16,17} + P_{17}\, \pi_{17,16} + P_{18}\, \pi_{18,15} + P_{19}\, \pi_{19,14} + P_{20}\, \pi_{20,13} + P_{21}\, \pi_{21,12} + P_{22}\, \pi_{22,11} + P_{23}\, \pi_{23,10} + P_{24}\, \pi_{24,9} + P_{25}\, \pi_{25,8} + P_{26}\, \pi_{26,7} + P_{27}\, \pi_{27,6} + P_{28}\, \pi_{28,5} + P_{29}\, \pi_{29,4} + P_{30}\, \pi_{30,3} + P_{31}\, \pi_{31,2} + P_{32}\pi_{32,1} + P_{33}\, \pi_{33,0}$$

$P_{33} = (P_o + P_1)\ \pi_{1,33} + P_2\ \pi_{2,32} + P_3\ \pi_{3,31} + P_4\ \pi_{4,30} + P_5\ \pi_{5,29} + P_6\ \pi_{6,28} + P_7\ \pi_{7,27} + P_8\ \pi_{8,26} + P_9\ \pi_{9,25} + P_{10}\ \pi_{10,24} + P_{11}\ \pi_{11,23} + P_{12}\ \pi_{12,22} + P_{13}\ \pi_{13,21} + P_{14}\ \pi_{14,20} + P_{15}\ \pi_{15,19} + P_{16}\ \pi_{16,18} + P_{17}\ \pi_{17,17} + P_{18}\ \pi_{18,16} + P_{19}\ \pi_{19,15} + P_{20}\ \pi_{20,14} + P_{21}\ \pi_{21,13} + P_{22}\ \pi_{22,12} + P_{23}\ \pi_{23,11} + P_{24}\ \pi_{24,10} + P_{25}\ \pi_{25,9} + P_{26}\ \pi_{26,8} + P_{27}\ \pi_{27,7} + P_{28}\ \pi_{28,6} + P_{29}\ \pi_{29,5} + P_{30}\ \pi_{30,4} + P_{31}\ \pi_{31,3} + P_{32}\pi_{32,2} + P_{33}\ \pi_{33,1} + P_{34}\ \pi_{34,0}$

$P_{34} = (P_o + P_1)\ \pi_{1,34} + P_2\ \pi_{2,33} + P_3\ \pi_{3,32} + P_4\ \pi_{4,31} + P_5\ \pi_{5,30} + P_6\ \pi_{6,29} + P_7\ \pi_{7,28} + P_8\ \pi_{8,27} + P_9\ \pi_{9,26} + P_{10}\ \pi_{10,25} + P_{11}\ \pi_{11,24} + P_{12}\ \pi_{12,23} + P_{13}\ \pi_{13,22} + P_{14}\ \pi_{14,21} + P_{15}\ \pi_{15,20} + P_{16}\ \pi_{16,19} + P_{17}\ \pi_{17,18} + P_{18}\ \pi_{18,17} + P_{19}\ \pi_{19,16} + P_{20}\ \pi_{20,15} + P_{21}\ \pi_{21,14} + P_{22}\ \pi_{22,13} + P_{23}\ \pi_{23,12} + P_{24}\ \pi_{24,11} + P_{25}\ \pi_{25,10} + P_{26}\ \pi_{26,9} + P_{27}\ \pi_{27,8} + P_{28}\ \pi_{28,7} + P_{29}\ \pi_{29,6} + P_{30}\ \pi_{30,5} + P_{31}\ \pi_{31,4} + P_{32}\pi_{32,3} + P_{33}\ \pi_{33,2} + P_{34}\ \pi_{34,1} + P_{35}\ \pi_{35,0}$

Normative condition is $\sum_{k=0}^{34} P_k = 1$.

To solve given system of equations I need to define coefficients $\pi_{k,j}$. Since I consider that the operating time is constant, the coefficients will be calculated from the relationship

$$\pi_{k,j} = \binom{m-k}{j}\left(1 - e^{-\lambda\tau}\right)^j \left(e^{-\lambda\tau}\right)^{m-k-j}$$

where
m = 35 machines
$\lambda = 0.6$
$\tau = 0.12$ h
k = 1, 2, 3, ...,35
j = 0, 1, 2, 3, . . . ,35
The programme Excel was used to calculate coefficients $\pi_{k,j}$.
The coefficients are presented in Table 3.

Table 3. **Part 1 Coefficients $\pi_{k,j}$**

	j							
k	0	1	2	3	4	5	6	7
1	0.084805	0.217027	0.269534	0.2164	0.126233	0.057009	0.02074	0.006244
2	0.091188	0.226499	0.272773	0.212157	0.119766	0.052285	0.018365	0.005332
3	0.098052	0.236167	0.275528	0.207387	0.113171	0.047702	0.016157	0.004517
4	0.105432	0.246007	0.27775	0.202091	0.106478	0.043278	0.014116	0.003795
5	0.113367	0.255991	0.279388	0.196273	0.099719	0.03903	0.012241	0.003159
6	0.121901	0.266084	0.280389	0.189941	0.092928	0.034973	0.01053	0.002604
7	0.131076	0.276246	0.80701	0.18311	0.08614	0.031122	0.00898	0.002124
8	0.140942	0.28643	0.28027	0.175797	0.079392	0.027488	0.007586	0.001713
9	0.15155	0.296582	0.279042	0.168026	0.072721	0.024084	0.006345	0.001364
10	0.162957	0.30664	0.276965	0.159826	0.066164	0.020916	0.005248	0.001072
11	0.175223	0.316532	0.273987	0.151233	0.059761	0.017993	0.004289	0.00083
12	0.188412	0.326175	0.270059	0.142289	0.05355	0.015316	0.003459	0.000632
13	0.202593	0.335477	0.265135	0.133043	0.047566	0.012889	0.002749	0.000473
14	0.217842	0.344331	0.259174	0.123549	0.041847	0.010709	0.00215	0.000347
15	0.234239	0.352618	0.252141	0.11387	0.036426	0.008774	0.001651	0.000249
16	0.25187	0.360201	0.244007	0.104075	0.031334	0.007075	0.001243	0.000174
17	0.270828	0.366928	0.234755	0.094239	0.0266	0.005606	0.000914	0.000118
18	0.291213	0.372627	0.224377	0.084443	0.022246	0.004353	0.000655	7.75E-05
19	0.313132	0.377105	0.212882	0.074776	0.018292	0.003304	0.000456	4.9E-05
20	0.336701	0.380146	0.200292	0.065328	0.014752	0.002443	0.000306	2.97E-05
21	0.362044	0.381509	0.186652	0.056196	0.011632	0.001751	0.000198	1.7E-05
22	0.389295	0.380923	0.17203	0.047478	0.008934	0.00121	0.000121	9.14E-06
23	0.418596	0.378087	0.15652	0.03927	0.006651	0.000801	7.03E-05	4.54E-06
24	0.450104	0.372666	0.140251	0.03167	0.004767	0.000502	3.78E-05	2.03E-06
25	0.483982	0.364288	0.123388	0.024766	0.003262	0.000295	1.85E-05	7.95E-07
26	0.520411	0.352537	0.10614	0.018641	0.002105	0.000158	7.95E-06	2.56E-07
27	0.559582	0.336952	0.088767	0.013363	0.001257	7.57E-05	2.85E-06	6.13E-08
28	0.601701	0.317025	0.071586	0.00898	0.000676	3.05E-05	7.66E-07	8.24E-09
29	0.64699	0.292189	0.054982	0.005518	0.000311	9.38E-06	1.18E-07	
30	0.695688	0.261818	0.039413	0.002967	0.000112	1.8E-06		
31	0.748052	0.22522	0.025428	0.001276	2.4E-05			
32	0.804357	0.181629	0.013671	0.000343				
33	0.8649	0.1302	0.0049					
34	0.93	0.07						

Table 3. **Part 2 Coefficients πk,j**

	j								
k	8	9	10	11	12	13	14	15	16
1	0.001586	0.000345	6.49E-05	1.07E-05	1.54E-06	1.96E-07	2.21E-08	2.22E-09	1.98E-10
2	0.001304	0.000273	4.93E-05	7.75E-06	1.07E-06	1.3E-07	1.4E-08	1.33E-09	1.13E-10
3	0.001062	0.000213	3.69E-05	5.56E-06	7.32E-07	8.48E-08	8.66E-09	7.82E-10	6.26E-11
4	0.000857	0.000165	2.73E-05	3.92E-06	4.92E-07	5.41E-08	5.24E-09	4.47E-10	3.36E-11
5	0.000684	0.000126	1.99E-05	2.72E-06	3.24E-07	3.38E-08	3.09E-09	2.48E-10	1.75E-11
6	0.000539	9.47E-05	1.43E-05	1.85E-06	2.09E-07	2.06E-08	1.77E-09	1.33E-10	8.78E-12
7	0.00042	7.02E-05	1E-05	1.24E-06	1.32E-07	1.22E-08	9.85E-10	6.92E-11	4.23E-12
8	0.000322	5.12E-05	6.94E-06	8.07E-07	8.1E-08	7.04E-09	5.3E-10	3.45E-11	1.95E-12
9	0.000244	3.67E-05	4.7E-06	5.14E-07	4.84E-08	3.92E-09	2.74E-10	1.65E-11	8.54E-13
10	0.000182	2.58E-05	3.11E-06	3.19E-07	2.8E-08	2.11E-09	1.36E-10	7.51E-12	3.53E-13
11	0.000133	1.78E-05	2.01E-06	1.92E-07	1.57E-08	1.09E-09	6.44E-11	3.23E-12	1.37E-13
12	9.52E-05	1.19E-05	1.26E-06	1.12E-07	8.42E-09	5.36E-10	2.88E-11	1.3E-12	4.9E-14
13	6.67E-05	7.81E-06	7.65E-07	6.28E-08	4.33E-09	2.51E-10	1.21E-11	4.87E-13	1.6E-14
14	4.57E-05	4.97E-06	4.48E-07	3.38E-08	2.12E-09	1.1E-10	4.75E-12	1.67E-13	4.7E-15
15	3.04E-05	3.05E-06	2.53E-07	1.73E-08	9.76E-10	4.52E-11	1.7E-12	5.12E-14	1.2E-15
16	1.96E-05	1.8E-06	1.36E-07	8.36E-09	4.2E-10	1.7E-11	5.49E-13	1.38E-14	2.59E-16
17	1.22E-05	1.02E-06	6.92E-08	3.79E-09	1.66E-10	5.78E-12	1.55E-13	3.12E-15	4.4E-17
18	7.29E-06	5.49E-07	3.31E-08	1.58E-09	5.96E-11	1.72E-12	3.71E-14	5.58E-16	5.25E-18
19	4.15E-06	2.78E-07	1.46E-08	6.01E-10	1.88E-11	4.36E-13	7.04E-15	7.06E-17	3.32E-19
20	2.23E-06	1.31E-07	5.9E-09	2.02E-10	5.07E-12	8.8E-14	9.46E-16	4.75E-18	
21	1.12E-06	5.62E-08	2.12E-09	579E-11	1.09E-12	1.26E-14	6.78E-17		
22	5.16E-07	2.16E-08	6.5E-10	1.33E-11	1.67E-13	9.69E-16			
23	2.13E-07	7.14E-09	1.61E-10	2.21E-12	1.38E-14				
24	7.65E-08	1.92E-09	2.89E-11	1.98E-13					
25	2.24E-08	3.75E-10	2.82E-12						
26	4.83E-09	4.04E-11							
27	576E-10								

Table 3. **Part 3 Coefficients πk,j**

	j								
k	17	18	19	20	21	22	23	24	25
1	1.58E-11	1.12E-12	7.12E-14	4.02E-15	2.02E-16	8.97E-18	3.52E-19	1.22E-20	3.66E-22
2	8.5E-12	5.69E-13	3.38E-14	1.78E-15	8.3E-17	3.41E-18	1.23E-19	3.85E-21	1.04E-22
3	4.43E-12	2.78E-13	1.54E-14	7.54E-16	3.24E-17	1.22E-18	3.99E-20	1.13E-21	2.72E-23
4	2.23E-12	1.31E-13	6.73E-15	3.04E-16	1.2E-17	4.1E-19	1.21E-20	3.03E-22	6.39E-24
5	1.08E-12	5.9E-14	2.8E-15	1.16E-16	4.16E-18	1.28E-19	3.35E-21	7.36E-23	1.33E-24
6	5.05E-13	2.54E-14	1.11E-15	4.16E-17	1.34E-18	3.67E-20	8.41E-22	1.58E-23	2.38E-25
7	2.25E-13	1.03E-14	4.1E-16	1.39E-17	3.98E-19	9.53E-21	1.87E-22	2.93E-24	3.53E-26
8	9.5E-14	3.97E-15	1.42E-16	4.26E-18	1.07E-19	2.2E-21	3.59E-23	4.51E-25	4.07E-27
9	3.78E-14	1.42E-15	4.51E-17	1.19E-18	2.56E-20	4.37E-22	5.72E-24	5.39E-26	3.24E-28
10	1.41E-14	4.71E-16	1.31E-17	2.95E-19	5.29E-21	7.23E-23	7.1E-25	4.45E-27	1.34E-29
11	4.84E-15	1.42E-16	3.37E-18	6.34E-20	9.09E-22	9.33E-24	6.11E-26	1.92E-28	
12	1.52E-15	3.81E-17	7.55E-19	1.14E-20	1.22E-22	8.36E-25	2.74E-27		
13	4.26E-16	8.91E-18	1.41E-19	1.59E-21	1.14E-23	3.91E-26			
14	1.04E-16	1.74E-18	2.07E-20	1.56E-22	5.59E-25				
15	2.13E-17	2.68E-19	2.12E-21	7.98E-24					
16	3.44E-18	2.88E-20	1.14E-22						
17	3.89E-19	1.63E-21							
18	2.33E-20								

Table 3. **Part 4 Coefficients πk,j**

	j								
k	26	27	28	29	30	31	32	33	34
1	9.54E-24	2.13E-25	4E-27	6.23E-29	7.82E-31	7.59E-33	5.36E-35	2.44E-37	5.41E-40
2	2.41E-24	4.71E-26	7.59E-28	9.86E-30	9.89E-32	7.21E-34	3.39E-36	7.73E-39	
3	5.5E-25	9.21E-27	1.24E-28	1.28E-30	9.67E-33	4.7E-35	1.1E-37		
4	1.11E-25	1.55E-27	1.66E-29	1.29E-31	6.5E-34	1.58E-36			
5	1.92E-26	2.15E-28	1.73E-30	8.98E-33	2.25E-35				
6	2.76E-27	2.31E-29	1.24E-31	3.22E-34					
7	3.07E-28	1.71E-30	4.6E-33						
8	2.36E-29	6.57E-32							
9	9.39E-31								

Source: Own processing

To simplify the probability solution, I have modified the normalization condition:

$$\sum_{k=0}^{34} P_k = 1$$

$$\frac{\sum_{k=0}^{34} P_k}{P_0} = \frac{1}{P_0}$$

A fraction $\frac{P_k}{P_0}$ is substituted by coefficient q_k, so I get a relationship

$$\sum_{k=0}^{34} q_k = \frac{1}{P_0}$$

The substitution has to be done also in system equations:

$P_o = (P_o + P_1)\ \pi_{1,0}$

$P_{o\,=}\ P_o.\ \pi_{1,0} + P_1.\ \pi_{1,0}$

$1 = \pi_{1,0} + \frac{P1}{P_0}\ \pi_{1,0}$ where $\frac{P1}{P_0} = q_1$

Then $q_1 = \frac{1-\pi_{1,0}}{\pi_{1,0}}$.

By gradual adjustments of equations and pitching, I get successive coefficients q_k:

$$q_2 = \frac{q_1(1-\pi_{1,1})-\pi_{1,1}}{\pi_{2,0}}$$

$$q_3 = \frac{q_2(1-\pi_{2,1})-q_1\pi_{1,2}-\pi_{1,2}}{\pi_{3,0}}$$

$$q_4 = \frac{q_3(1-\pi_{3,1})-q_2\pi_{2,2}-q_1\pi_{1,3}-\pi_{1,3}}{\pi_{4,0}} \qquad \text{etc.}$$

Values q_k calculated by using these expressions are shown in Tab. 4.

Table 4. **Values qk for 35 BUZI machines**

k		k		k	
1	10.79179035	13	2030121595	25	30751415476
2	90.28225125	14	5115489265	26	14366684717
3	679.7969011	15	11517280481	27	5496044301
4	4667.217752	16	23095292520	28	1688668522
5	29206.80674	17	41102421662	29	406425839.9
6	166376.1608	18	64664024256	30	74090428.74
7	861493.714	19	89532458829	31	9753168.462
8	4048461.125	20	1.08553E+11	32	861269.6823
9	17237255.01	21	1.14597E+11	33	44786.76772
10	66372683.93	22	1.0465E+11	34	1010.96671
11	230666982.6	23	82044576283		
12	721952563.5	24	54736136090		
				∑	7.55473E+11

Source: Own processing

Now I can calculate P_0 from the relationship $\sum_{k=0}^{34} q_k = \frac{1}{P_0}$, from which it follows that $P_0 = \frac{1}{\sum_{k=0}^{34} q_k}$

Therefore, P_0 = 1.32367E-12. I can further express the other conditional probabilities of P_k under condition that there enters the request into the system, provided that there are already in the system k requirements that are not necessary for my solution to the problem, but for completeness they will be introduced. These are listed in Table 5.

Table 5. **Conditional probabilities P_k for 35 BUZI machines**

k	P_k	k	P_k	k	P_k
0	1.32367E-12	12	0.000955629	24	0.072452771
1	1.42848E-11	13	0.002687218	25	0.040704833
2	1.19504E-10	14	0.006771237	26	0.019016799
3	8.99829E-10	15	0.015245119	27	0.007274968
4	6.17787E-09	16	0,030570626	28	0.002235246
5	3.86603E-08	17	0,054406185	29	0.000537975
6	2.20228E-07	18	0.085594053	30	9.80715E-05
7	1.14034E-06	19	0.11851174	31	1.291E-05
8	5.35884E-06	20	0.143689127	32	1.14004E-06
9	2.28165E-05	21	0.151689441	33	5.92831E-08
10	8.78558E-05	22	0.138521858	34	1.33819E-09
11	0.000305328	23	0.108600228		
				∑	1

Source: Own processing

Such probabilities can be interpreted by, for example, the probability of entering the twentieth machine into the system, i.e. it needs repair, while there are already 19 machines in the system (one is being repaired and the others are waiting) as P_{20} =0.143689127.

The mean value of machine waiting before repairs is estimated with P_0 from relationship

$$EW = (m - 1)\,\tau - \frac{1 - P_0}{\lambda} = (35 - 1).\ 0.12 - \frac{1 - 1{,}32367E - 12}{0{,}6} =$$
2.413333 hours/machine.

The middle cycle of each machine is comprised of a mean value of:
1.67 hour operating outside the system, i.e. the machine is working,
2.41 hour waiting for repair,
0.12 hour duration of the repair,
which is a total of 4.2 h. This figure represents the mean duration of the cycle.

The mean number of returns per machine is expressed from relationship $N\,[\frac{1}{\lambda} + EW + \tau] = 1$, from which it follows that N = 0.238. It must be taken into account that m = 35 and hence the repair per hour needs an average

35 x N = 35x 0.238 = 8.33 machines. The length of one fix is τ = 0.12 h. This means that one worker – performing operation works 0.12 x 8.33

= 0.9996 h/h. The worker is utilized at 99.96%, while only 0.04% of his time is unused.

On the other hand, there is a non-productive machine lag, which is calculated as the share of its waiting for repair and the mean duration of the cycle:

Non-productive machine lag $= \frac{2{,}41}{4{,}2} = 0.5738$, i.e. 57.38%.

Conclusion

From the enclosed M / G / 1 system model created for the Svitex knitting company, it is clear that the non-productive lag of the BUZI machine is distinctive, while the worker is used to the maximum. In addition, it is known that the worker performs other activities, in addition to repairing malfunctions, such as machine reworking, quality control, and continuous machine maintenance. This means that the waiting time of the machines in the queue is still prolonging.

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